Real-Time Simulations in 3D Multibody Mechanical Systems with Impact, Contact and Friction

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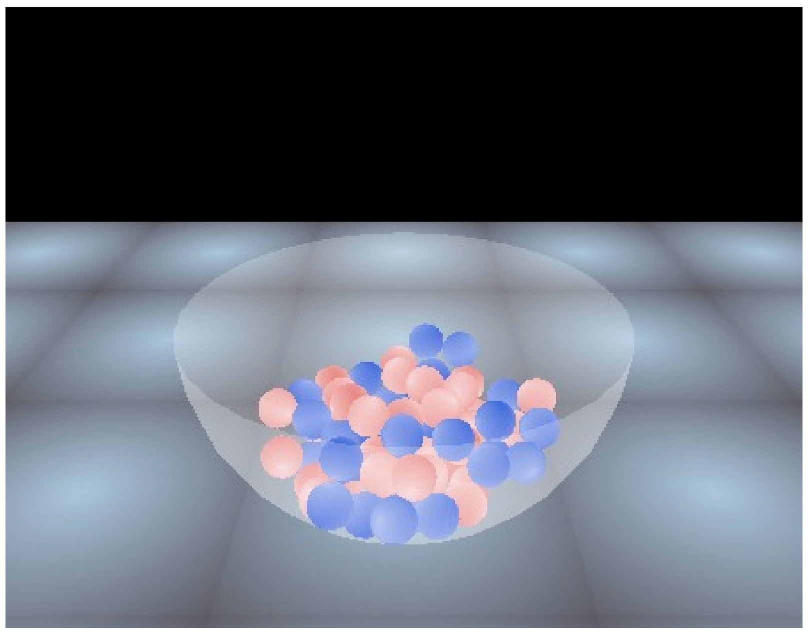
Topics...

- **Motivations** *Computer Graphics & Computational Mechanics*
- **Formulation** *Non Smooth Contact Dynamics*
- **Algorithms** *Iterative & direct methods*
- **Results** *Real-Time & Mechanical Constraint*

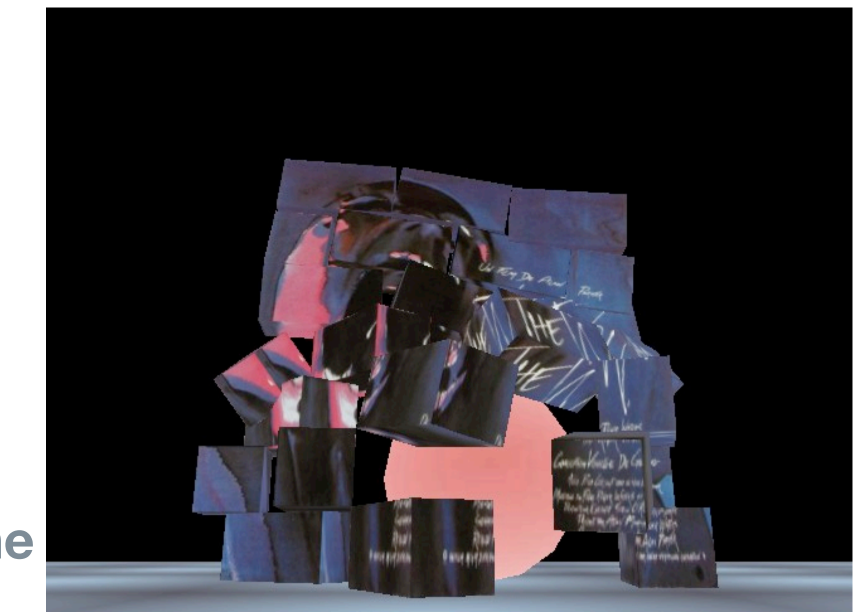
Motivations

Computer Graphics / Robotics / Computational mechanics

Overview

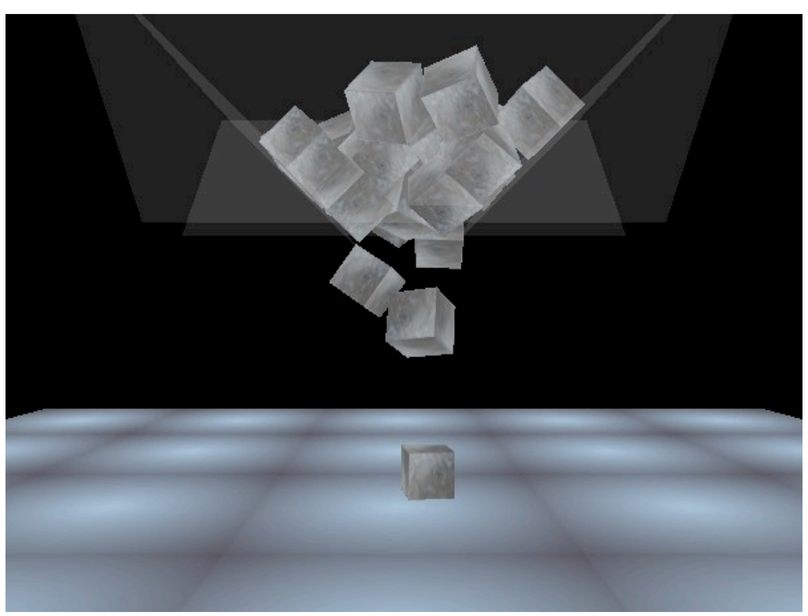


Dense packing *Nb contact > Nb bodies*

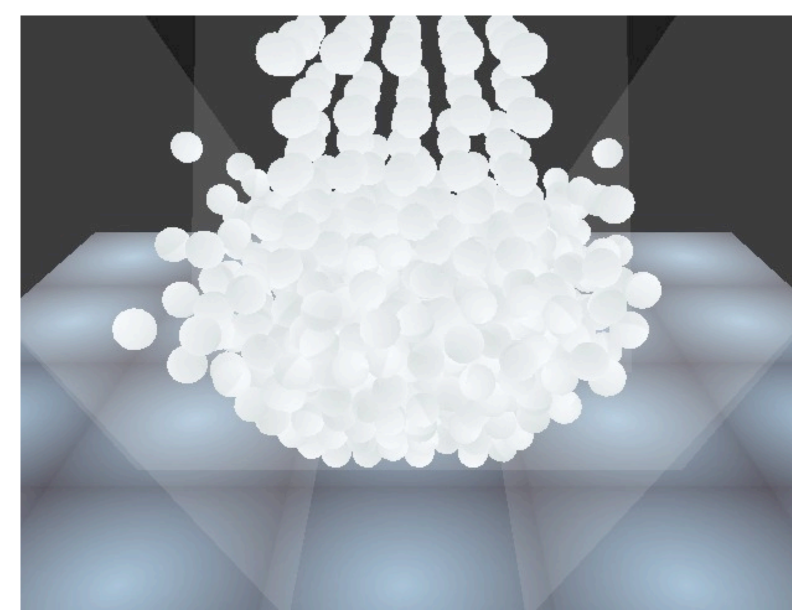


From static configuration to static one

... and reciprocally



In view to integrate haptic and pseudo-haptic interfaces



RELATED WORKS

Compliant models / Impulse Based / Constraint Based / ...

Event-Driven time integration scheme

One Step Non Smooth Problem:
- Direct methods or explicit resolution
- Approximation of the friction cone

OBJECTIVES

Unified approach for impact and contact
Time-stepping time integration scheme
Iterative methods

Formulation

Non Smooth Contact Dynamics approach

Framework of J. J. Moreau & M. Jean

Based on Equation of motion

$$\mathbb{M}(\mathbf{q})\ddot{\mathbf{q}} = \mathbf{F}(t, \mathbf{q}, \dot{\mathbf{q}}) + \sum_{\alpha} \mathbf{R}_{\alpha}$$

$\mathbb{M} \in \mathbb{R}^{6n_b \times 6n_b}$	Inertia matrix
$\mathbf{q} \in \mathbb{R}^{6n_b}$	Configuration parameter
$\mathbf{F}(t, \mathbf{q}, \dot{\mathbf{q}}) \in \mathbb{R}^{6n_b}$	External and internal forces
$\mathbf{R}_{\alpha} \in \mathbb{R}^{6n_b}$	Contact forces

Reformulated in terms of a measure differential equation

$$\mathbb{M}(\mathbf{q})d\dot{\mathbf{q}} = \mathbf{F}(t, \mathbf{q}, \dot{\mathbf{q}})dt + d\mathbf{R}$$

Definition of the **local frame**

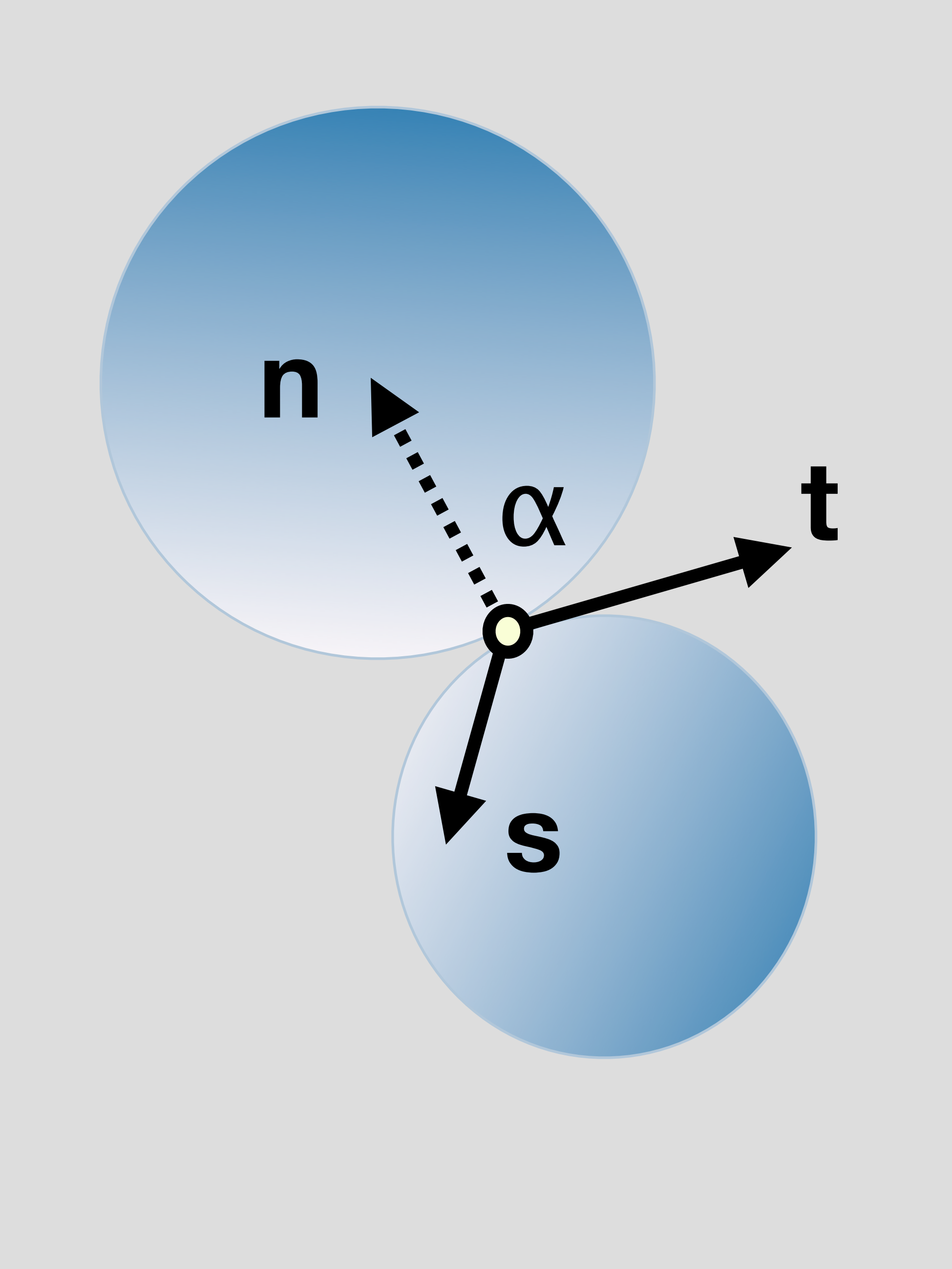
$\mathbf{r}_\alpha$ Contact Impulse

$\mathbf{u}_\alpha$ Relative velocity between particles

Local variables related to the generalized variables via linear mapping

$$\mathbf{R}_\alpha = \mathbb{H}_\alpha(\mathbf{q})\,\mathbf{r}_\alpha$$

$$\mathbf{u}_\alpha = \mathbb{H}^*_\alpha(\mathbf{q})\,\dot{\mathbf{q}}$$



Contact, Friction and Impact Law

Velocity Signorini Condition

$$\text{If} \quad g \leq 0 \quad \text{then} \quad u_n \geq 0, \quad r_n \geq 0, \quad u_n.r_n = 0 \tag{6}$$

Moreau's Impact Law

$$\text{If} \quad g \leq 0 \quad \text{then} \quad \begin{aligned} & u_n^+ + e_n u_n^- \geq 0, \quad r_n \geq 0, \quad (u_n^+ + e_n u_n^-).r_n = 0 \\ & \mathbf{u}_t^+ + e_t \mathbf{u}_t^- \geq 0, \quad \mathbf{r}_t \geq 0, \quad (\mathbf{u}_t^+ + e_t \mathbf{u}_t^-).\mathbf{r}_t = 0. \end{aligned} \tag{7}$$

Coulomb's Friction Law

$$\begin{cases} \text{If} \quad ||\mathbf{u}_t|| = 0, \quad ||\mathbf{r}_t|| \leq \mu r_n \\ \text{If} \quad ||\mathbf{u}_t|| \neq 0, \quad ||\mathbf{r}_t|| = \mu r_n, \, \mathbf{u}_t = -\kappa \mathbf{r}_t, \kappa \geq 0 \end{cases} \tag{8}$$

Frictional contact problem formulation

Equations of motion are reformulated in the local frame

We define $\mathbb{W} = \mathbb{H}^* \mathbb{M}^{-1} \mathbb{H}$, call the Delassus operator

$$\begin{cases} \mathbb{W}\mathbf{r}_{i+1} - \mathbf{u}_{i+1} = -\mathbf{u}_{free} \\ law_\alpha[\mathbf{u}_{\alpha,i+1}, \mathbf{r}_{\alpha,i+1}] = .true., \quad \alpha = 1, \ldots, n_c \end{cases}$$

with law_α synthetic local frictional contact law

Algorithms

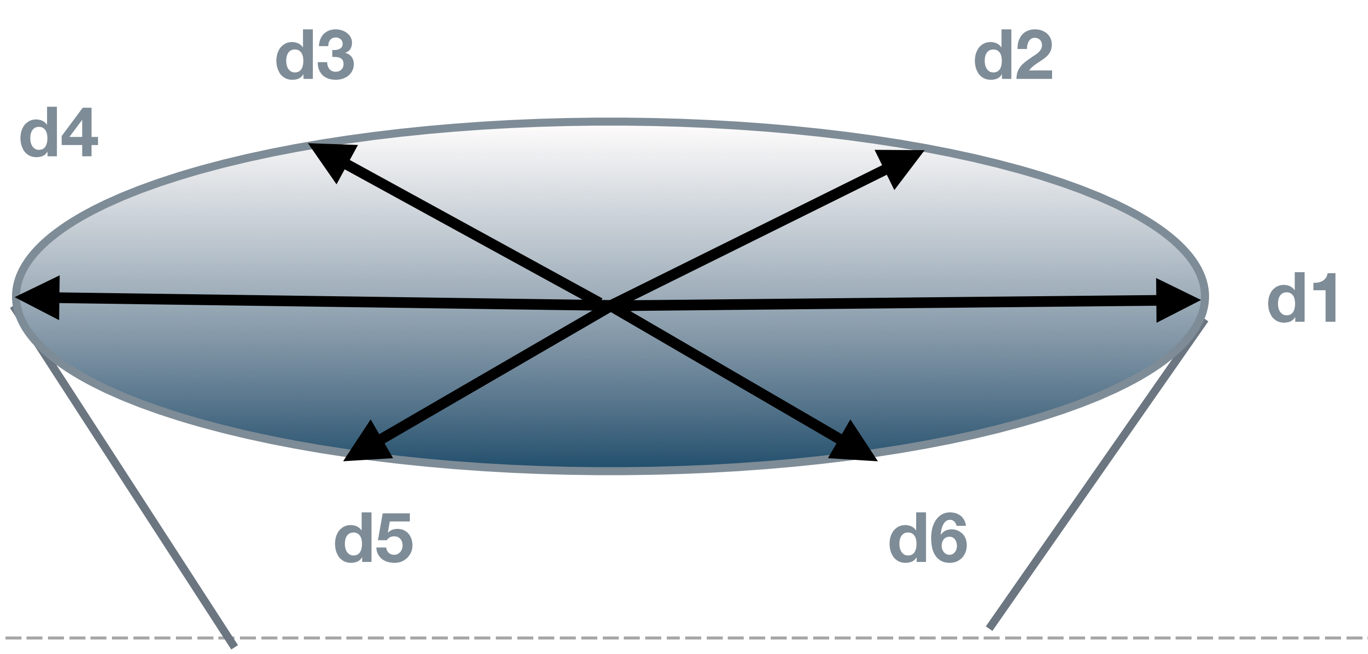
Direct and Iterative Methods

Approximation of the Friction Cone

Klarbring (1986) / Al-Fahed *et al* (1991) / Stewart (1996)

Polyhedral cone define as

$$\widehat{FC}(\mathbf{q}) = \{ r_n \mathbf{n} + \mathbf{D}\boldsymbol{\beta} \,|\, r_n \geq 0,\ \beta \geq \mathbf{0},\ \mathbf{e}^T\boldsymbol{\beta} \leq \mu r_n \}$$



with $\mathbf{D} = [\mathbf{d}_1, ...\mathbf{d}_\nu]$

Lemke's Algorithm

Pivot method (direct method) based on a LCP formulation

Frictionless case

$$\mathbf{0} \leq \mathbf{r} \perp \mathbb{W}\mathbf{r} + \mathbf{u}_{free} \geq \mathbf{0}$$

Frictional case

$$\mathbf{0} \leq \tilde{\mathbf{r}} \perp \tilde{\mathbb{W}}\tilde{\mathbf{r}} + \tilde{\mathbf{u}}_{free} \geq \mathbf{0}$$

Basic determination of the pivot

Quadratic Programming QLD (Schittkowski 1987)

Direct method based on the minimization of a quadratic function

$$\begin{array}{ll} \text{minimize} & \mathbf{q}(\mathbf{z}) = \frac{1}{2}\mathbf{z}^T \mathbb{Q}\mathbf{z} + \mathbf{p}^T\mathbf{z} \\ \text{subject to} & \mathbb{C}\mathbf{z} - \mathbf{b} \geq \mathbf{0} \end{array}$$

In the frictionless case $\mathbb{Q} = \mathbb{W}$, $\mathbb{C} = \mathbb{I}$ and $\mathbf{b} = \mathbf{0}$

In the frictional case $\mathbb{Q} = \mathbb{W} + \mathbb{W}^T$,

$$\mathbb{A} = \begin{bmatrix} \mathbb{I} \\ \mathbb{W} \end{bmatrix} \quad \text{and} \quad \mathbf{b} = \begin{bmatrix} \mathbf{0} \\ -\mathbf{u}_{free} \end{bmatrix}$$

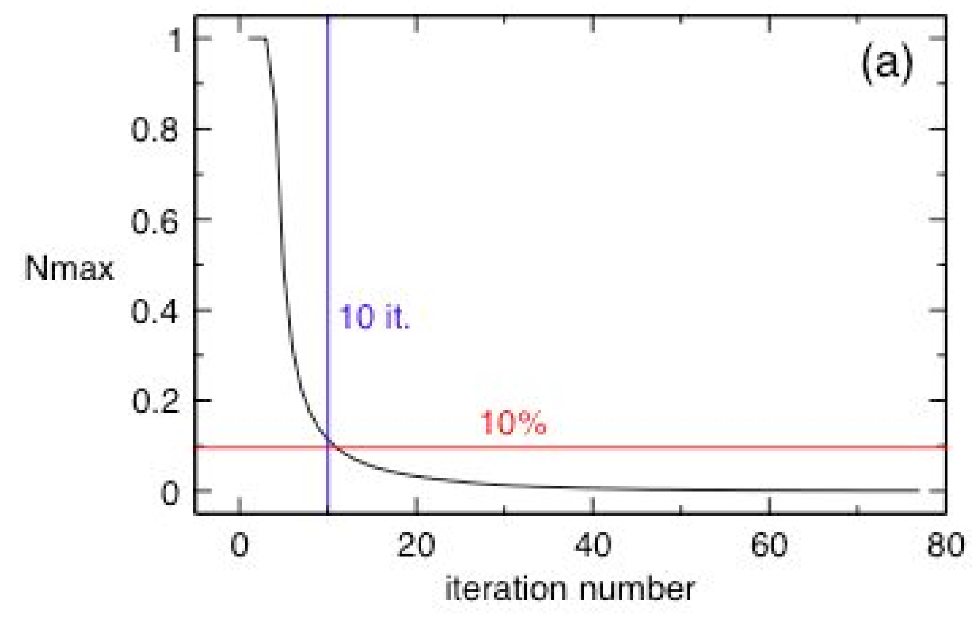
Non Smooth Gauss-Seidel algorithm

Iterative method based on a splitting scheme

Resolution contact by contact

$$\begin{cases} \mathbf{u}_\alpha^{k+1} - \mathbb{W}_{\alpha\alpha}\mathbf{r}_\alpha^{k+1} = \mathbf{u}_{\alpha,free} + \dots \\ \qquad\qquad \sum_{\beta<\alpha} \mathbb{W}_{\alpha\beta}\mathbf{r}_\beta^{k+1} + \sum_{\beta>\alpha} \mathbb{W}_{\alpha\beta}\mathbf{r}_\beta^{k} \\ law_\alpha(\mathbf{u}_\alpha^{k+1}, \mathbf{r}_\alpha^{k+1}) = .true. \end{cases}$$

Logarithmic convergence



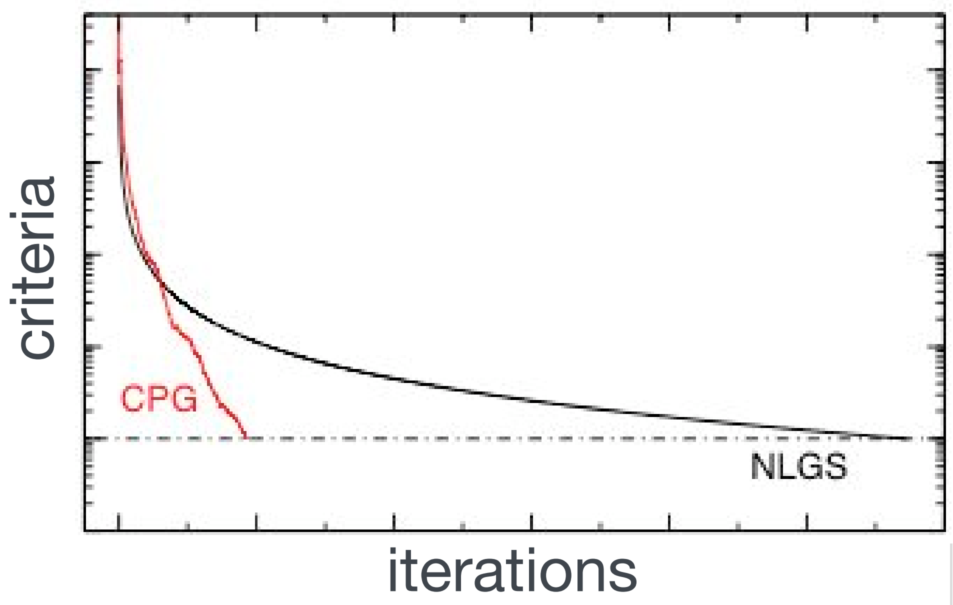
Conjugate Projected Gradient Renouf & Alart (2004)

Iterative method based on descent method with a Global Resolution based on a quasi-variational formulation

$$\mathbf{r} \in \underset{\tilde{\mathbf{r}} \in \mathcal{C}(\mu r_n)}{\operatorname{argmin}} \ \frac{1}{2}\tilde{\mathbf{r}}.\mathbb{W}\tilde{\mathbf{r}} - \mathbf{b}.\tilde{\mathbf{r}}$$

Iterative scheme

$$\mathbf{r}^{k+1} = \text{proj}\{\mathbf{r}^k + \alpha^{k+1}\{\text{proj}(-\mathbf{u}^k) + \beta\,\text{proj}(\mathbf{p}^k)\}\}$$



PATH solver

Ferris, Dirkse & Mudson (1993)

Iterative Newton method

Pseudo Lemke for the research of the descent direction

Based on :

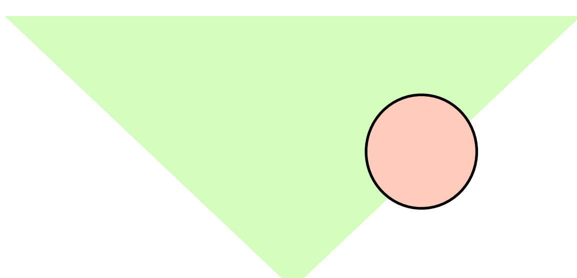
- a MCP formulation for the frictionless problem
- a MNCP formulation for the frictional contact problem using the formulation proposed by Glocker (1999).

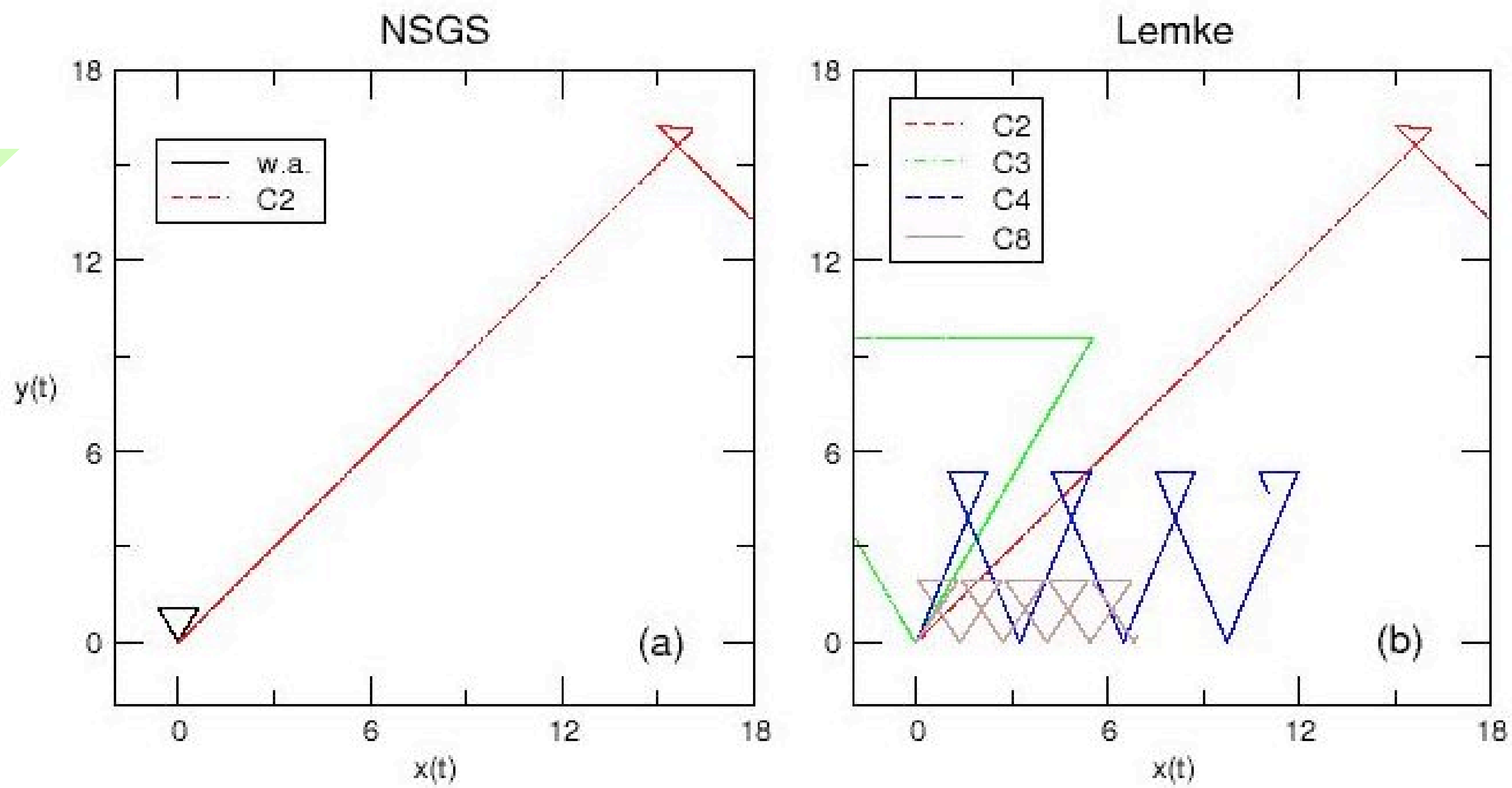
Simulations

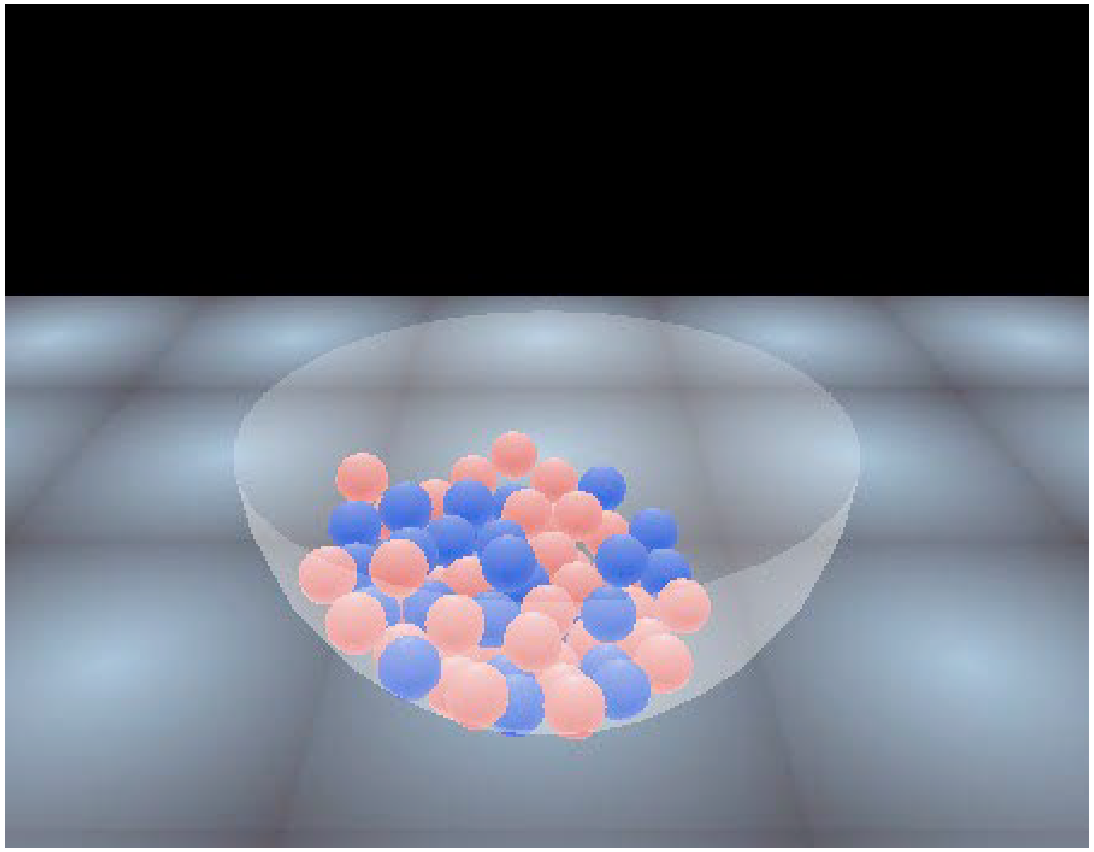
Results and Comments

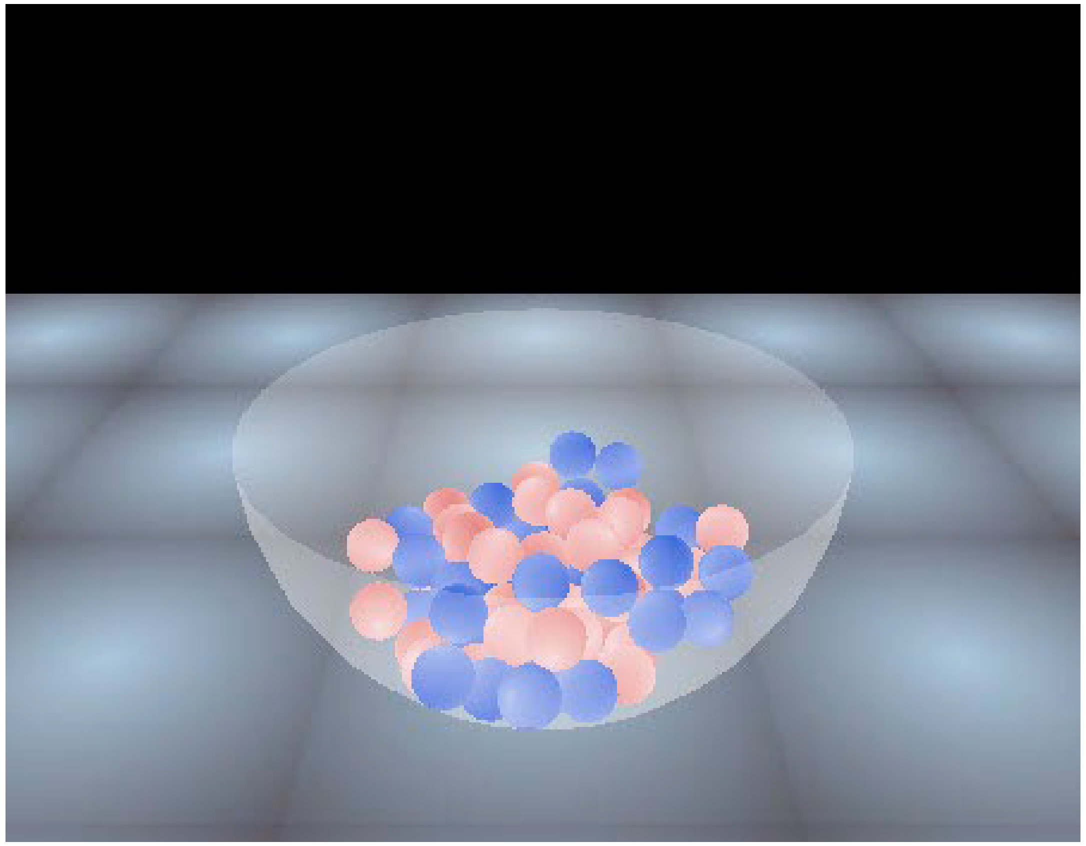
Pyramidal cone weaknesses

Trajectory of a ball driven by force









Frictionless simulation

Solver	Real Time ratio	Violation max %
Lemke	NO	NO
QP	0.35	1
NLGS	1.	4
CPG	0.35	1
PATH	0.5	8

Sphere packing

Frictional simulation:
NSGS algorithm

Nb Bodies	Nb Contact	Real Time ratio
80	160	>1
160	450	>1
320	1250	0.8
800	3200	0.2

Virtual masonry

Conclusions

* Unified approach with respect of real-time constraint for nb $\leq$ 200

* Efficiency of iterative algorithms

* Underline the importance of the Smooth Friction Cone Feature for the integration in haptic feedback software

Future works

* Implementation of a Generalized Newton Method [Alart 1993]

* Implementation of Lemke for PSD matrices [Cottle, Pang & Stone 1992]

* Integration in Haptic Feedback models

* Optimization of Collision Detection