How not to spoil frictional contact algorithms

The following “advices” will be highlighted in this talk.

1/ Don’t try to offer dynamical schemes said to be the most accurate, such as higher order algorithms.
2/ If bodies are deformable and dynamics is not too wild, use gap Signorini, else use velocity Signorini.
3/ If bodies are rigid, use velocity Signorini or inelastic shock.
4/ Estimate the gap at the right moment.
5/ If the instants of shocks may be carefully computed, higher order algorithms may be used, together with special procedures at the instant of shocks.
6/ Don’t trust graphs, write relations.
7/ Uniqueness is not the rule in rigid bodies collections. The algorithm operates is own choice.

I shall use basic features of:

the *LMGC90* software, the *Non Smooth Contact Dynamics* method

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Main purpose :

/ Software devoted to dynamics of large collection of divided materials;

/ Bodies are rigid or deformable without any restriction on behaviour (for instance, coupling multi physics);

/ Interaction laws may be of various kind & complexity. Signorini's unilateral condition and Coulomb's law are basic interactions between bodies;

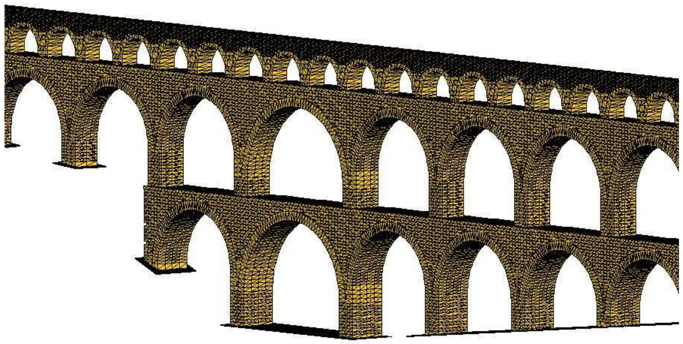
Special features :

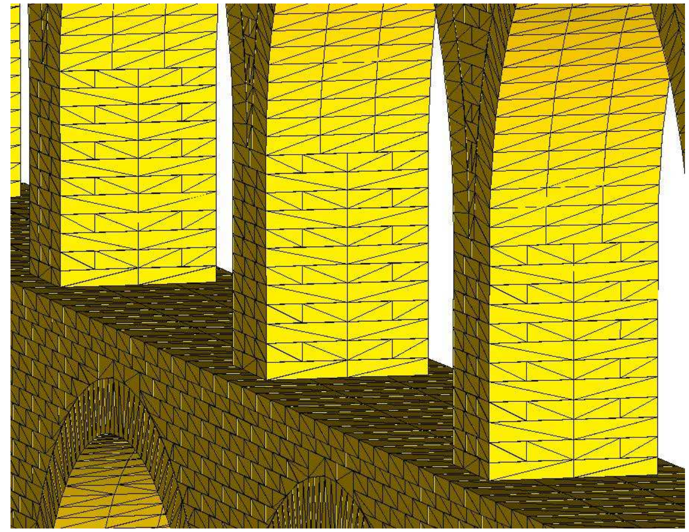
/ Non Smooth Mechanics implicit methods are used to solve the frictional contact problem;

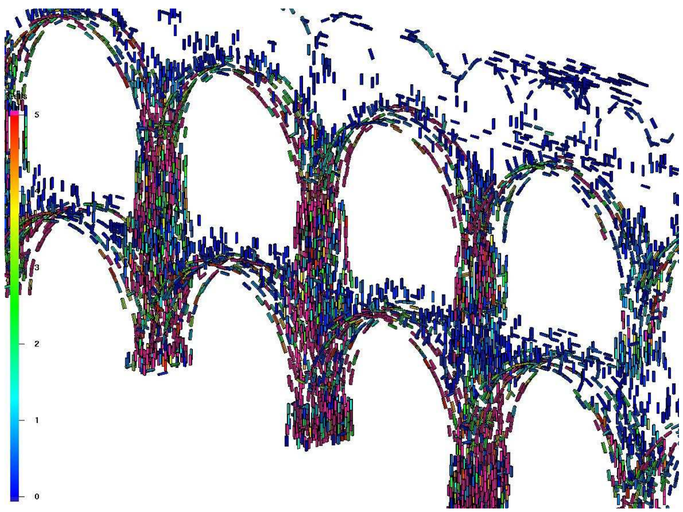
/ Implicit algorithm schemes are used for the dynamical equation;

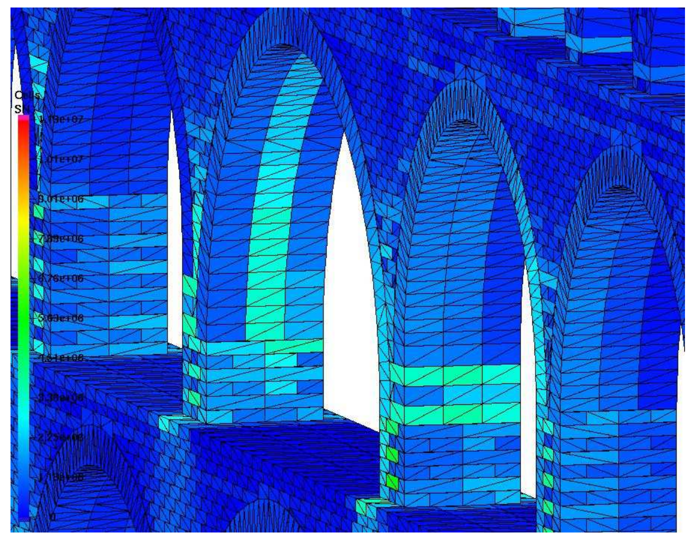
△ Deformable bodies, if any, are described by **finite element** methods.

See a few examples hereafter.



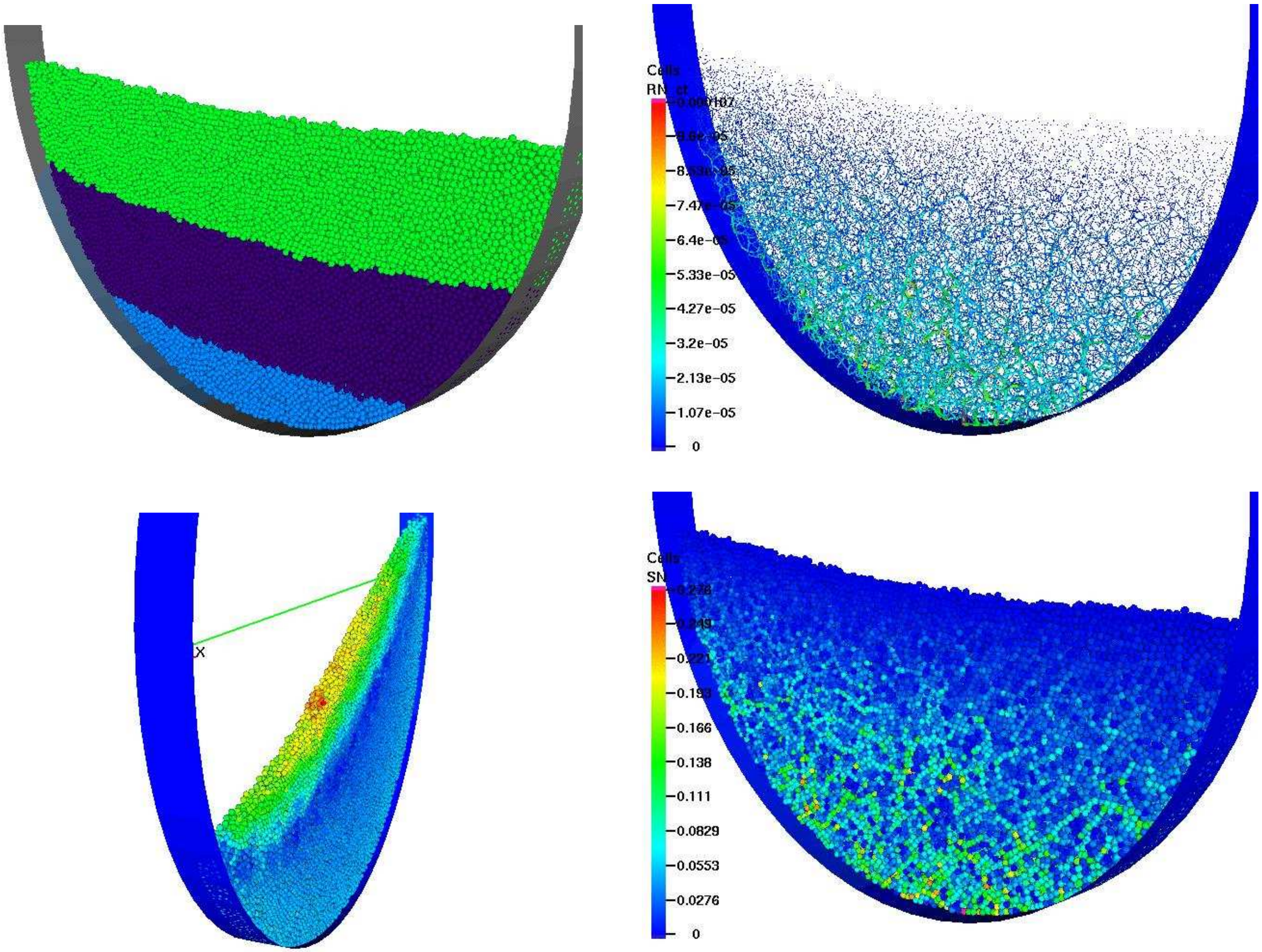






Masonry: Pont du Gard, 35000 blocks, 250000 contacts, author Brahim Chetouane

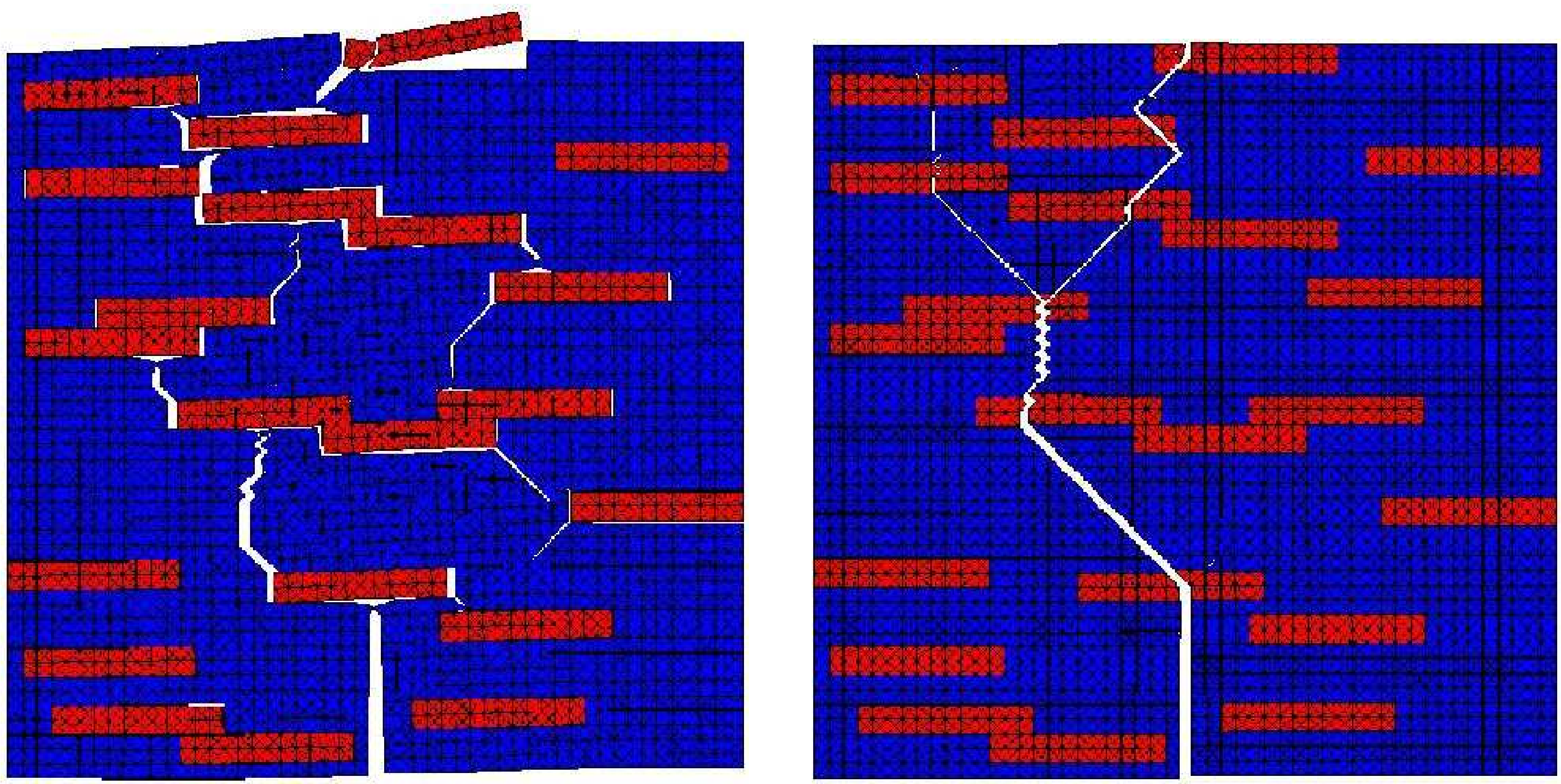
down left: main stress chains; down right: pressure colored blocks, high pressure pale blue, low pressure dark blue.



Granular Material : 19000 spheres rotating drum, author M. Renouf

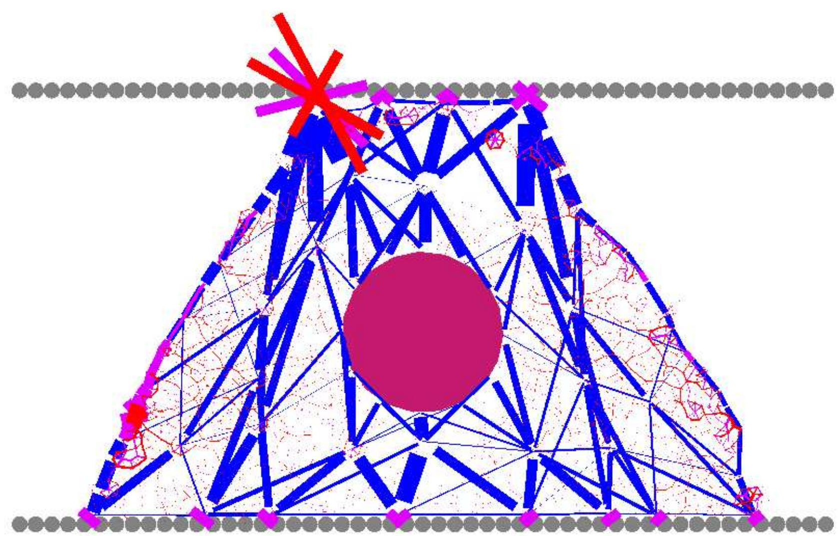
down right: velocities of the grains, up left: chain of reaction forces, down left: pressure in the grains.

NSCD



Micromechanic Application : dynamical cracks, weak vs strong zyrcaloy/hydrure interaction,

Authors F. Dubois, Y. Monnerie & al.



Biomechanic : granular tensegrities, a model of bone cell, author J.L. Milan

Adaptative nets; up: compressed cell, down: extended cell; red: compressed net, blue: tensile net.

The dynamical equation

$$M\ddot{q} = F(q,\dot{q}) + P(t) + r \ ,$$

q : vector of generalized variables describing degrees of freedom, coordinates of the center of gravity, rotations, of rigid bodies, nodes of meshed bodies...

M : mass matrix;

$F(q,\dot{q})$: internal forces;

$P(t)$: external forces;

r : representative of reactions.

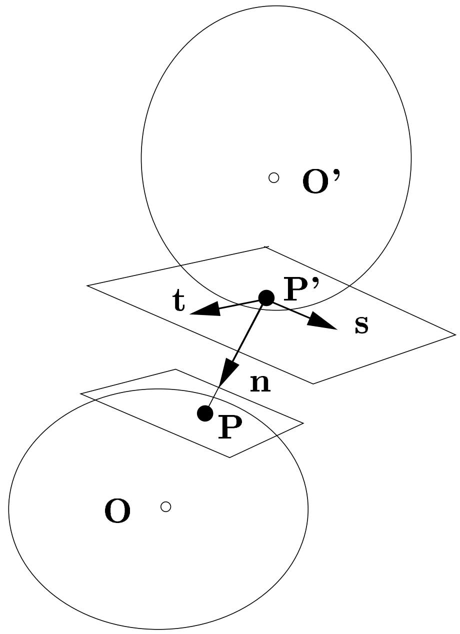
Shocks are expected. A relevant choice is:

q is a locally bounded variation function, r is a density of reaction forces or impulses.

The above equation is to be understood in the sense of distributions, or rewritten as a measure differential equation.

$$M d\dot{q} = F(q,\dot{q})dt + P(t)dt + r d\theta \ .$$

Local variables



/ n is the normal vector directed along $P'P$, where P, P', are proximal points;
/ $U = (U_T, U_N)$ is the relative velocity, U^-, U^+, left and right velocity;
/ $R = (R_T, R_N)$ is the reaction from O' onto O;
/ g is the gap, the signed distance $\overline{P'P}$.

Relations connecting local variables to generalized variables, at some candidate to contact labelled α:

$$U^\alpha = H^{*\alpha}(q)\,\dot{q} \quad , \quad r^\alpha = H^\alpha(q)\,R^\alpha \ . \tag{1}$$

$H^\alpha(q), H^{*\alpha}(q)$: transposed linear mappings.

$$\dot{g}^\alpha = U_N^\alpha \ . \tag{2}$$

The normal relative velocity is the time derivative of the gap function.

Unilateral conditions

* The classical form of the non penetration condition is,

$$\text{for all } t \geq t_0 \quad g(t) \geq 0.$$

* Another equivalent form is (Moreau):

$$g(t_0) \geq 0\,, \text{ for all } t \geq t_0\,, \text{ if } g(t) \leq 0\,, \text{ then } U_N^+(t) \geq 0\,.$$

* The classical Signorini condition is (*gap Signorini*),

$$g(t) \geq 0 \qquad R_N(t) \geq 0 \qquad g(t)\, R_N(t) = 0\,.$$

* An alternative form is (*velocity Signorini*),

$$g(t_0) \geq 0\,,$$
$$\text{if } g(t) > 0\,, \text{ then } R_N(t) = 0\,,$$
$$\text{if } g(t) \leq 0\,, \text{ then } U_N^+(t) \geq 0 \quad R_N(t) \geq 0 \quad U_N^+(t)\, R_N(t) = 0\,.$$

Velocity Signorini implies gap Signorini. The converse is almost true.

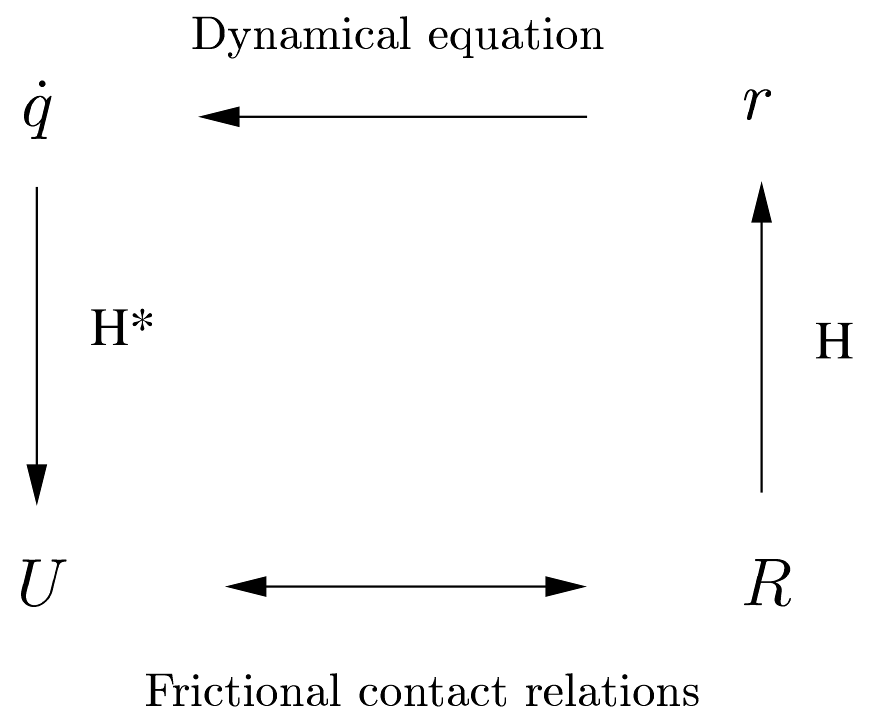
* The *inelastic shock* law writes:

$$g(t_0) \geq 0\,,$$
$$\text{if } g(t) > 0\,, \text{ then } R_N(t) = 0\,,$$
$$\text{if } g(t) \leq 0\,, \text{ then } U_N^+(t) = 0 \quad R_N(t) \geq 0.$$

The inelastic shock law implies velocity Signorini. The converse is almost true.

* Glocker proposes a complementarity condition using $\dot{U}_N$.

A choice of unilateral condition is offered. What are the pros and cons?



The structure of the equations of a dynamical frictional contact problem.

Discrete forms of the dynamical equation

Integrating both sides of the dynamical equation on an elementary subinterval $]t_i, t_{i+1}]$ of length h yields:

$$\left\{ \begin{array}{l} M(\dot{q}(t_{i+1}) - \dot{q}(t_i)) = \int_{t_i}^{t_{i+1}} F(q, \dot{q})\, ds + \int_{t_i}^{t_{i+1}} P(s)\, ds + \int_{]t_i, t_{i+1}]} r d\nu, \\ q(t_{i+1}) = q(t_i) + \int_{t_i}^{t_{i+1}} \dot{q} ds. \end{array} \right.$$

/ The mean value impulse $\frac{1}{h} \int_{]t_i, t_{i+1}]} r d\nu$ emerges as a fundamental unknown.

/ A numerical scheme is a choice of formula approximating integral terms.

/ Since shocks are expected, within the interval $]t_i, t_{i+1}]$ higher order (> 1) schemes are unrelevant: See advice 1.

Discrete forms of Signorini Coulomb relations

The proposed discrete form of the gap Signorini condition is,

$$g(i+1) \geq 0 \qquad R_N(i+1) \geq 0 \qquad g(i+1)\, R_N(i+1) \;=\; 0\,,$$

The proposed discrete form of the velocity Signorini condition is,

if contact is not forecasted, then

$$R_N(i+1) = 0\,,$$

if some contact is forecasted, then

$$U_N^+(i+1) \geq 0 \quad R_N(i+1) \geq 0 \quad U_N^+(i+1)\, R_N(i+1) \;=\; 0\,.$$

The proposed discrete form for Coulomb law is:

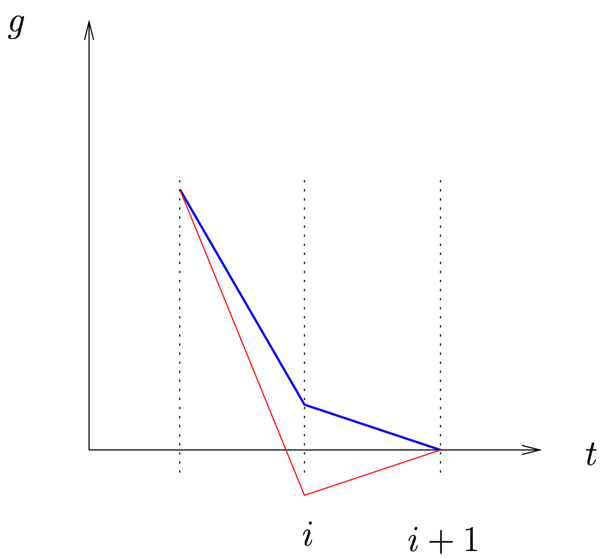
$$R_T(i+1) \in \mathrm{D}(\mu R_N(i+1))$$
$$\forall S \in \mathrm{D}(\mu R_N(i+1)) \quad (S - R_T(i+1))\, U_T(i+1) \geq 0\,,$$

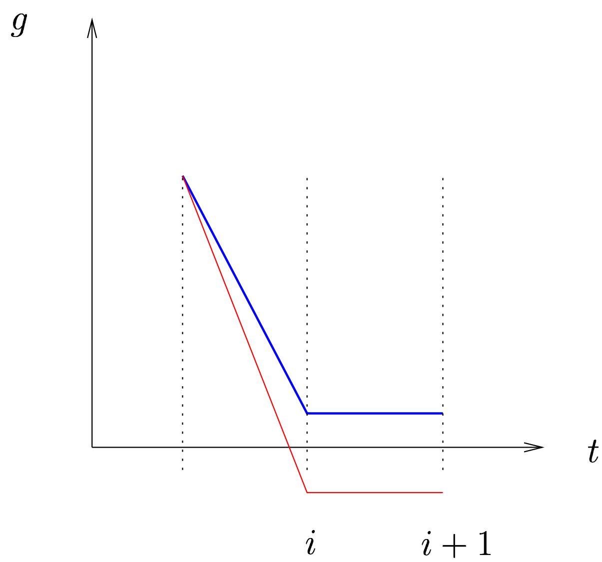
Shortly referred to as (α is some index labelling a candidate for contact)

$$\mathrm{SignCoul}\,(i, U^\alpha(i+1), R^\alpha(i+1))\,.$$

A full implicit approach is used. Primary variables: the mean value impulse; the relative velocity at the end of the time step (acting as U^+), the gap at the end of the time step.

A free particle is falling down on a rigid plane. Here are two possible computed trajectories: blue, a safe trajectory; red, an inaccuracy has occured at time i, so that the particle has penetrated.





gap Signorini
$g(i+1) = 0$
$U_N(i+1) = -g(i)/h$

* **Quasistatics,**
* **deformable bodies**
- masses of contacting nodes are negligible
- time step h is large
- $mU_N(i+1) = -mg(i)/h$ are negligible artefact impulses
- gap self correcting

* **Dynamics,**
* **rigid bodies**
- masses of contacting bodies are not negligible
- time step h is small
- $mU_N(i+1) = -mg(i)/h$ are significant artefact restoring impulses

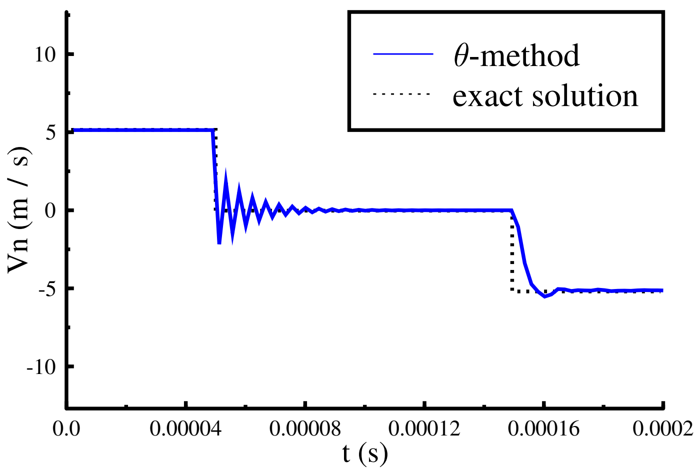
velocity Signorini
$g(i+1) = g(i)$
$U_N(i+1) = 0$

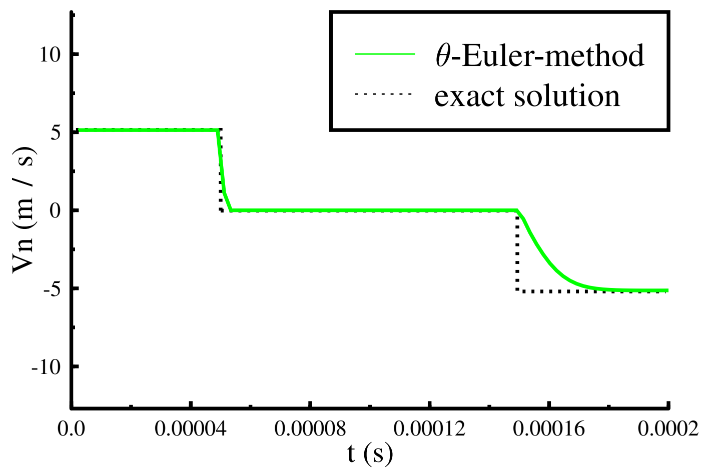
- fine from the point of view of impulses
- penetration should be controlled

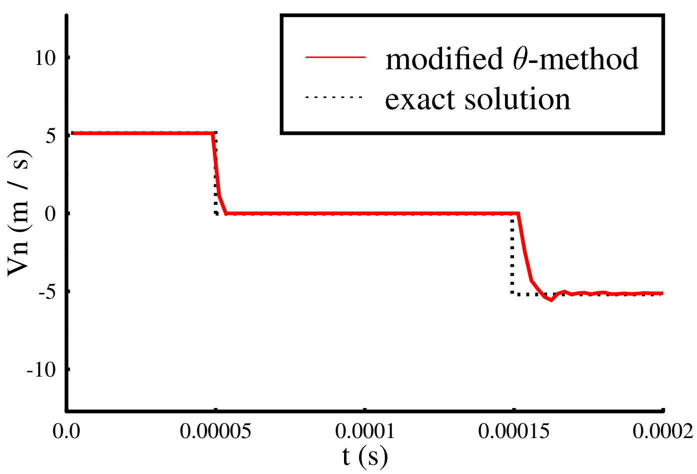
See advices 2,3

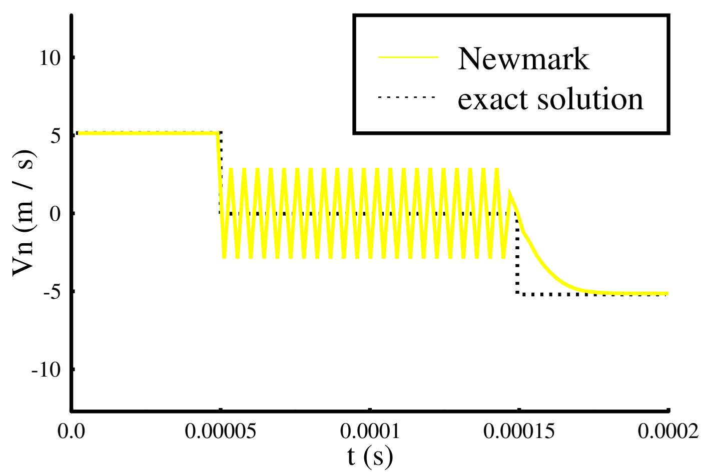
Colliding bars

Two colliding unidimensional symmetric elastic bars. Velocity of the left end point of the left bar versus time. Author D. Vola & al.









Why so spoiled?

A θ method approximation is adopted:

$$q(i+1) = q(i) + h(1-\theta)U_N(i) + h\theta U_N(i+1)\,, \quad (3)$$
$$0 < \theta \leq 1\,.$$

The gap Signorini condition is,

$$g \geq 0 \qquad R_N \geq 0 \qquad g\,R_N \;=\; 0\,,$$
$$\dot{g} = U_N\,.$$

A discrete form is,

$$g(i+1) \geq 0 \qquad R_N(i+1) \geq 0 \qquad g(i+1)\,R_N(i+1) \;=\; 0\,,$$
$$g(i+1) = g(i) + h(1-\theta)U_N(i) + h\theta U_N(i+1)\,. \qquad (4)$$

The approximation 4 is suggested by 3.

The results are the following, previous slide:
Newmark yellow , artefact oscillations.
θ method ($\theta = 0.55$) blue , artefact oscillations.

Explanation: For simplicity sake be lucky: $g(i) = 0$, prescribed: $g(i+1) = 0$, so that,

$$U_N(i+1) = -\frac{1-\theta}{\theta}U_N(i)\,.$$

The approximation 4 generates an artefact restitution law.

Another discrete form is obtained using an approximation of the gap at time $t_{i+1} + h(1-\theta)$.

$$g(i+1+1-\theta) \geq 0 \quad R_N(i+1) \geq 0 \quad g(i+1+1-\theta)\,R_N(i+1) \;=\; 0\,,$$
$$g(i+1+1-\theta) = g(i+1-\theta) + hU_N(i+1)\,.$$

The result is the following, previous slide:
θ method, with above relevant Signorini condition red.

See advice 4.

Event driven methods

$$\left\{ \begin{array}{l} M(\dot q(t_{i+1}) - \dot q(t_i)) = \int_{t_i}^{t_{i+1}} F(q,\dot q)\,ds + \int_{t_i}^{t_{i+1}} P(s)\,ds + \int_{]t_i,t_{i+1}]} r d\nu, \\ q(t_{i+1}) = q(t_i) + \int_{t_i}^{t_{i+1}} \dot q ds. \end{array} \right.$$

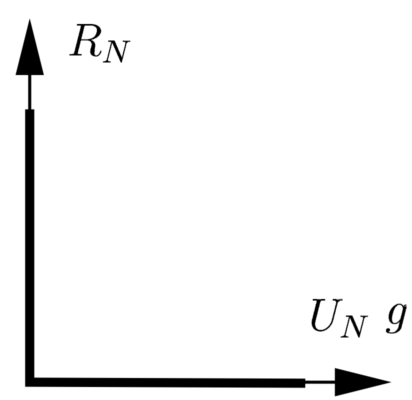
/ If shocks are not expected in the subinterval $]t_i, t_{i+1}]$ higher order algorithms may be used. Special procedures to deal with shocks are to be used at the instants of shocks. These event driven methods may thus be more precise during free flight periods. Detecting the instants of shocks or changes in the friction status may be unfeasable when two many contacts are under consideration. The implicit method should be preferred.

Advice 5.

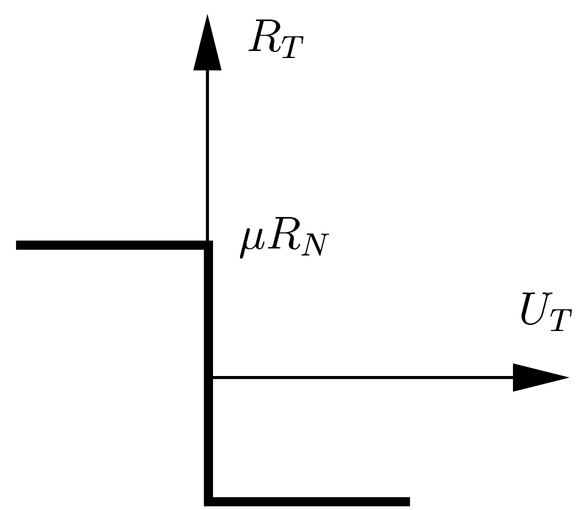
Pictural model

Graphs are not sufficient to carry all the informations. Gap Signorini condition is described correctly by the hereafter graph; but such a graph cannot carry the full information for the velocity Signorini condition. Even more so with Coulomb static dynamic friction law, cohesion laws, etc.

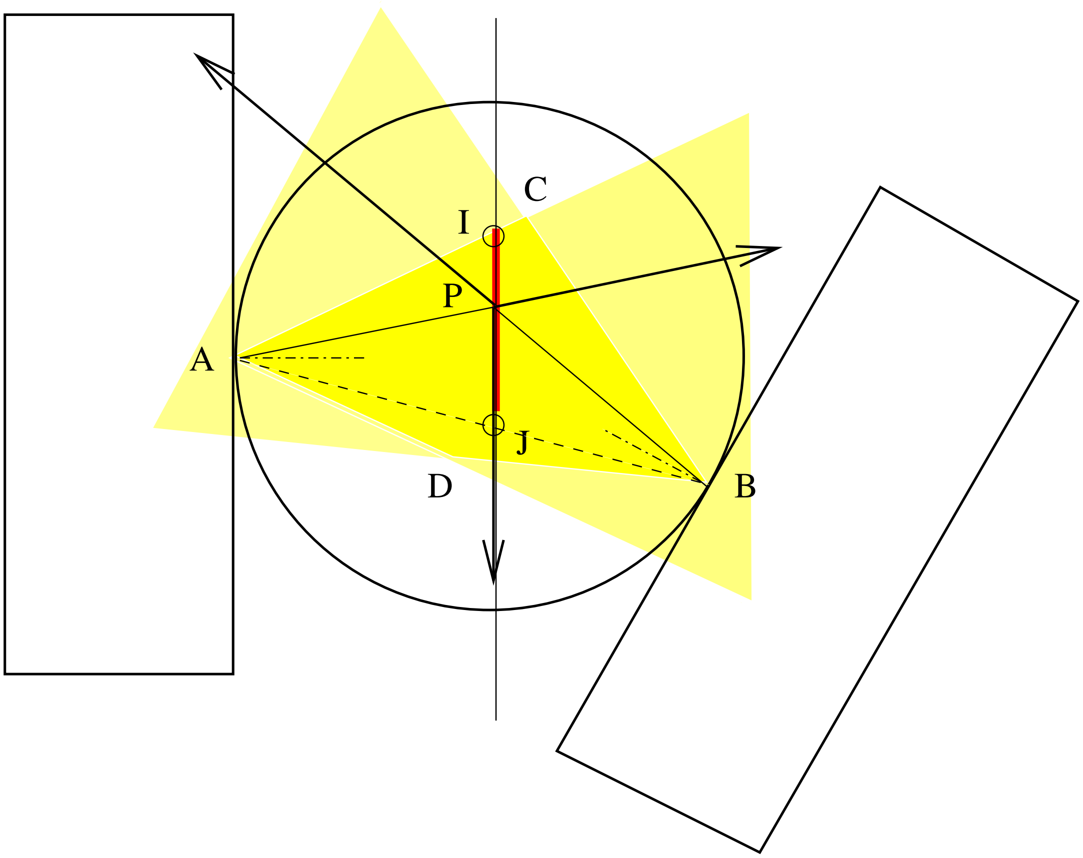
See advice 6



Signorini graph



Coulomb graph



Static equilibrium of a rigid disk wedged between two rigid planes, under gravity, with Coulomb friction (Coulomb cones are yellow coloured); an example of non uniqueness of the reactions. Any point P on the segment IJ is a possible meeting point for the gravity force and the reaction forces exerted on the disk at contact points A, B.

See warning 7

[1] JEAN, M.: *The Non Smooth Contact Dynamics Method.* in Computer Methods in Applied Mechanics and Engineering, special issue on computational modeling of contact and friction, J.A.C. Martins and A. Klarbring editors, 177, (1999) 235-257.

[2] JOURDAN, F., JEAN, M., ALART, P.: *An alternative method between implicit and explicitit scheme devoted to frictional contact problems in deep drawing simulation.* in Journal of Material Processing Technology, vol 80-81, pp 257-262, 1998.

[3] RADJAI, F., JEAN, M., MOREAU, J.J., ROUX, S.: *Force distributions in dense two-dimensional granular systems.* Physical Review Letters, Vol 77, n. 2, 8 July 1996.

[4] CAMBOU, B., JEAN, M.: *Micromécanique des matériaux granulaires.* Collection Méthodes de l'Ingénierie en Mécanique, hermès, 2001.

[5] JEAN, M.: *Stability of multibodies with frictional unilateral contact.* Multibody Dynamics 2005, Advances in Computational Multibody Dynamics, Madrid, 21-24 June 2005.